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GRAVITY VECTOR DETERMINATION FROM INERTIAL AND AUXILIARY DATA  
AND POTENTIAL UTILIZATION OF GENERATED VECTOR COMPONENT INFORMATION

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Narrative Summary

The determination of gravity anomalies and deflections of the vertical by means of initial and terminal gravity and astrogeodetic deflection data, and multiple inertial measurements has been pursued by the U.S. Army Engineer Topographic Laboratories (ETL) and the Geodetic Survey of Canada since about 1976. The inertial equipment employed has been the Rapid Geodetic Survey System (RGSS), developed by Litton Systems for ETL. This paper presents a symbiosis of pertinent in-house research involving inertial technology, mathematics, and physical geodesy, conducted in support of the Defense Mapping Agency, the U.S. Air Force, and the U.S. Army under consideration of potential utilization of generated data. Following an introduction, it first discusses the determination of gravity anomalies and the more detailed derivation of single horizontal channel deflection errors in semi-flat terrain and provides present and improved future accuracy estimates. Secondly, it presents a Wiener-type optimal solution for deflections of the vertical under application of the collocation method in physical geodesy with prior second order gyro bias removal. Thirdly, it outlines an advanced deflection estimation method under consideration of horizontal channel and gyro interactions and its modification in the case of strongly mountainous terrain. Fourthly, it summarizes the development of a partially associated optimal interpolation method for gravity anomalies and deflections of the vertical in mountainous terrain. Fifthly, it addresses potential civil and military applications including the construction of gravity anomaly and deflection networks from data obtained by multiple surveys, consideration of gravity vector data in geodetic network optimization, upward continuation of gravity anomalies and deflections of the vertical, and a gravity-programmed inertial positioning system. Sixthly, it comments on the need of a gradiometer-aided inertial system for rapid, high accuracy deflection determination, subterraneous mass detection, and the differential transformations between Cartesian and natural or astronomic coordinates. The paper's conclusion will include but will not be limited to the identification of supplemental and correlated research to achieve promising scientific and technological progress for civil and military applications.

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Gravity Vector Determination from Inertial and Auxiliary Data and  
Potential Utilization of Generated Vector Component Information

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1. INTRODUCTION. The present Rapid Geodetic Survey System (RGSS), developed by Litton Systems, Inc. for the U.S. Army Engineer Topographic Laboratories (ETL), has the capability of determining horizontal and vertical positions, deflections of the vertical, and gravity anomalies with average rms errors of 1 m, 0.3 m, 2 arcsec, and 2 mgal, respectively, for 50 km runs under utilization of post-mission adjustments. It operates as a quasi local-level system. It thus does not require altitude damping, permits Kalman stochastic error control without great complexity under consideration of observed velocity errors at vehicle stops, and employs effective post-mission adjustments with the aid of terminal position and gravity vector information only. Present critical hardware consists of an A-1000 vertical accelerometer, two A-200 horizontal accelerometers, and two G-300 gyroscopes. Although data utilization from repeated runs has resulted in average deflection rms errors of about 1 arcsec, the requirement of a maximum rms error of 0.5 arcsec necessitates the incorporation of higher performance gyroscopes and accelerometers. This would simultaneously achieve conventional surveying accuracy ( $10^{-5}$ ). Following the approximate elimination of the effects of constant gyro biases on positions under utilization of accurate initial and terminal coordinates, the remaining dominant error source is gyro correlated random noise. In this respect, the pertinent parameters for the G-300 instruments are a standard deviation of  $0.002^{\circ}$  hr<sup>-1</sup> and a correlation time of 3 hours. Another error source to be considered is the accelerometer scale factor, assumed to be constant for a test run. The standard scale factor error for the A-200 accelerometer is 0.01%. Correlated accelerometer noise is characterized by a standard deviation of 10 mgal and a correlation time of 40 minutes. Both correlated gyro random drift and correlated accelerometer measurement errors affect the accuracy of deflections of the vertical estimated from inertial data and initial and terminal deflection components, possibly augmented by corresponding geodetic azimuths. In order to achieve

deflection accuracies smaller than or equal to 0.5 arcsec rms, it is, therefore, necessary to install higher performance gyros and accelerometers. A significant reduction of the two autocorrelation parameters is also a necessary prerequisite for the optimal statistical estimation of deflections of the vertical. The Litton G-1200 gyro is expected to be compatible with the stringent deflection accuracy requirement. Its random drift error is about  $0.001^{\circ} \text{ hr}^{-1}$  and its autocorrelation parameter is short. For details, reference is made to Litton [1] and Huddle [2]. The A-1000 accelerometer addressed by Litton [3,4] has a standard scale factor error of approximately 0.005%. Correlated accelerometer noise is 2 mgal rms, equivalent to about 0.35 arcsec.

2. DETERMINATION OF VERTICAL GRAVITY COMPONENT. In a local Cartesian coordinate system tangent to a normal gravity surface, with x as west-east coordinate, y as south-north coordinate and z as vertical coordinate, positive upward, it is

$$\vec{g} = (g_n, -g\xi, -g) \quad (1)$$

In eq. (1),  $\vec{g}$  is the gravity vector at the origin,  $-g$  is its vertical component, and  $g_n$  and  $-g\xi$  are its horizontal components,  $\eta$  and  $\xi$  being the prime and meridian deflections, respectively. For computational and estimation purposes it is advantageous to use the gravity anomaly  $\Delta g = g - \gamma$  with  $\gamma$  as normal gravity, to be found by consideration of the elevation above the normal reference surface.

The RGSS error differential equation for the vertical channel, including first and second order time derivatives, is

$$\ddot{z} = 2\omega_s^2 z - (\rho_N + 2\Omega_N) \dot{x} - \rho_E \dot{y} + \Delta g + a_z + A_x \delta_{zx} - A_y \delta_{zy} \quad (2)$$

where the symbol  $\delta$  of position errors  $\delta x$ ,  $\delta y$ ,  $\delta z$  is omitted for simplicity. In eq. (2),  $\omega_s^2 = g/R^3$  with  $R$  as the earth's mean radius is the squared Schuler frequency,  $\rho_N = V_x R^{-1}$  with  $V_x$  as the system's east velocity is the north angular rate,  $\Omega_N$  is the north earth rate,  $\rho_E = -V_y R^{-1}$  with  $V_y$  as the system's north velocity is the east angular rate,  $a_z$  is the vertical accelerometer error,  $A_x$  and  $A_y$  are east and north accelerometer outputs, and  $\delta_{xz}$  and  $\delta_{yz}$  are z-axis misalignment errors in the x,z and y,z planes. At the system's stop, the term involving x may be estimated since x is observed, and the last two terms are omitted. At terrestrial vehicle speeds, the term involving y becomes insignificant.

1. See Litton [5], p. 3-132.

Because of initial calibration  $\ddot{z}(t_0) = \ddot{z}_0 = 0$  and  $z_0 = \dot{t}_0 = \dot{f}_0 = A_{x0} = A_{y0} = 0$ , the term  $\Delta g_0 + a_{z0}$  has to be subtracted from  $\ddot{z}(t_v) = \ddot{z}_v$ . At a survey vehicle stop, therefore, under omission of  $\rho_N \ddot{x}$  and  $\rho_E \ddot{y}$  in eq. (2),

$$\Delta g_v = \Delta g_0 + \ddot{z}_v + 2\Omega_N \dot{x}_v - 2\omega_s^2 z_v - (a_{z_v} - a_{z_0}), \quad (3)$$

where the error terms  $2\omega_s^2 z_v$  and  $a_{z_v} - a_{z_0}$  are weakly correlated. If  $z$  and  $a_z$  are considered as random variables with zero means and estimated variances and covariances, respectively, the gravity anomaly error variance is

$$\text{var } \delta g_v = 4\omega_s^4 \text{ var } z_v + \text{var } (a_{z_v} - a_{z_0}) \quad (4)$$

Since  $z_v$  has an average rms error of 0.3 m due to Kalman filter estimation by means of observed  $z_v$  at vehicle stops and a post-mission adjustment under utilization of a terminal  $z_e$ , the rms error induced by  $z_v$  is only 0.1 mgal. High accuracy determinations of gravity anomalies or of the vertical component of gravity by means of a local-level inertial system thus require a high performance vertical accelerometer, high accuracy height determinations and, possibly, utilization of  $\dot{x}$ -observations at vehicle stops.

The availability of  $\Delta g_e$  and  $z_e$  at the end of the survey permits the determination of  $a_{z_e} - a_{z_0} = f_e$  in eq. (3). Empirical corrections for

$a_{z_v} - a_{z_0}$  in the form

$$C_v = c_v (a_{z_e} - a_{z_0}) = c_v f_e \quad (5)$$

prevent error growths to about  $2 \text{ var } a_z$ . Under restrictive assumptions, with the omission of the subscript  $z$ ,

$$c_v = \text{cov } (a_v - a_0, a_e - a_0) \cdot [\text{var } (a_e - a_0)]^{-1} \quad (6)$$

**3. SINGLE HORIZONTAL CHANNEL DETERMINATION OF DEFLECTIONS OF THE VERTICAL.** While the determination of gravity anomalies may be less rapidly accomplished with high accuracy by means of gravimeter and height measurements, astrogeodetic and gravimetric determinations of

deflections of the vertical are time-consuming. According to Moritz [6], astrogeodetic and gravimetric rms errors are approximately 0.2 arcsec and 1.5 arcsec, respectively. Rapid inertial-astronomic determinations of  $\eta$  and  $\xi$  with rms errors not exceeding 0.35 arcsec would thus be highly economical and satisfactory for many purposes. Both the single and coupled horizontal channel determinations of  $\eta$  and  $\xi$  require the utilization of the simplified error differential equations<sup>2</sup>:

$$\ddot{x} = S_N \phi_z - g \phi_N + g \eta + a_E \quad (7)$$

$$\ddot{y} = -S_E \phi_z + g \phi_E - g \xi + a_N \quad (8)$$

$$\dot{\phi}_z = R^{-1} \tan \phi \cdot \dot{x} + R^{-1} (\Omega_N + \rho_N \sec^2 \phi) y - \omega_E \phi_N + \omega_N \phi_E + \alpha \quad (9)$$

$$\dot{\phi}_N = R^{-1} \dot{x} + \omega_E \phi_z - \omega_z \phi_E + \beta \quad (10)$$

$$\dot{\phi}_E = -R^{-1} \dot{y} - \omega_N \phi_z + \omega_z \phi_N + \gamma \quad (11)$$

Symbols used in the foregoing equations including total time derivatives of first and second order are, except for those defined in section 2,

$\phi_z$	azimuth platform attitude error
$\phi_N$	platform tilt error about north axis
$\phi_E$	platform tilt error about east axis
$S_E$	east acceleration of survey vehicle
$S_N$	north acceleration of survey vehicle
$a_E$	correlated east accelerometer error
$a_N$	correlated north accelerometer error
$\phi$	geographic latitude
$\omega_z$	$= \Omega_z + R^{-1} \tan \phi V_x$ vertical spatial rate
$\omega_N$	$= \Omega_N + \rho_N$ north spatial rate
$\omega_E$	$= -R^{-1} V_y$ east spatial rate
$\alpha$	azimuth axis angular drift rate error
$\beta$	north axis angular drift rate error
$\gamma$	east axis angular drift rate error

For land vehicles, eqs. (9) - (11) may be simplified by omission of  $\rho_N$ .

2. A more complex system may be required. See Litton [5], pp. 3-252 - 3-254.



$\omega_E$ , and  $\rho_z = R^{-1} \tan \phi \cdot V_x$ , and by use of constant accelerations  $S_N$  and  $S_E$  which should be approximately achieved. Then,  $\omega_z = \Omega \sin \phi$  and  $\omega_N = \Omega \cos \phi$  where  $\Omega$  denotes the earth's inertial angular velocity. The initial conditions at  $t_0 = 0$  are, under consideration of plumbline leveling,  $\omega_z(0) = 0$ ,  $\phi_N(0) = \eta_0$ ,  $\phi_E(0) = \xi_0$ ,  $x(0) = y(0) = \dot{x}(0) = \dot{y}(0) = 0$ .

In the general RGSS mode of operation, position and gravity vector information is generated. The Kalman filter mechanization is simplified by setting  $\omega_E = \omega_N = \omega_z = 0$  in eqs. (9) - (11) which results in the uncoupling of the horizontal channel differential equations. This is not critical in the context of present instrumentation for travel periods of 2 hours. As an example, eqs. (8) and (10) may then be formulated at the end of the first travel period, whence  $t = t_1$ ,

$$\ddot{y}_1 = g\phi_1 - g\xi_1 + a_{N_1} - a_{N_0} \quad (12)$$

$$\phi_{E_1} = -R^{-1}y + \int_0^{t_1} \gamma dt + \xi_0 \quad (13)$$

In eq. (12), accelerometer calibration at  $t_0 = 0$  has been taken into account. The quantity  $-R^{-1}y$  in eq. (13) is well estimated by the observed velocity error  $\gamma$ . After implementation of the corresponding tilt correction, under neglect of insignificant estimation errors and consideration of the bias error  $\bar{\phi}_E(t_1)$ ,

$$\ddot{y}(t_1) = \ddot{y}_1 = g\bar{\phi}_{E_1} - g(\xi_1 - \xi_0) + g \int_0^{t_1} (\gamma - \bar{\gamma}) dt + a_{N_1} - a_{N_0} \quad (14)$$

$$\ddot{y}(t_n) = \ddot{y}_n = g\bar{\phi}_{E_n} - g(\xi_n - \xi_0) + g \int_0^{t_n} (\gamma - \bar{\gamma}) dt + a_{N_n} - a_{N_0} \quad (15)$$

The propagation of constant gyro bias errors may be approximated by a solution of eqs. (9) - (11) with  $y = \dot{x} = \dot{y} = \omega_E = 0$  and by treating  $\omega_N$  and  $\omega_z$  as constants in accordance with Huddle [8]. The result is

$$\bar{\phi}_E = t(\bar{\gamma} + \bar{\beta}t\Omega_z/2 - \bar{\alpha}t\Omega_N/2) \quad (16)$$

$$\bar{\phi}_N = t(-\bar{\gamma}t\Omega_z/2 + \bar{\beta}) \quad (17)$$

$$\bar{\phi}_z = t(\bar{\gamma}t\Omega_N/2 + \bar{\alpha}) \quad (18)$$

Under consideration of terminal deflection and azimuth errors at  $t = t_n$

it is possible to estimate  $\bar{\alpha}$ ,  $\bar{\beta}$ ,  $\bar{\gamma}$  from

$$\begin{bmatrix} \bar{\phi}_E \\ \bar{\phi}_N \\ \bar{\phi}_Z \end{bmatrix}_{t_n} = \begin{bmatrix} \xi_n \\ \eta_n \\ A_n \end{bmatrix}_{\text{estimated}} - \begin{bmatrix} \xi_n \\ \eta_n \\ A_n \end{bmatrix}_{\text{observed}} \quad (19)$$

Equations (15) and (16) show that the computation of  $\bar{\phi}_{E_i}$  in eq. (14) is to the first order associated with an error

$$\delta \bar{\phi}_{E_i} = - \frac{t_i}{t_n} \left[ g \int_0^{t_n} (\gamma - \bar{\gamma}) dt + a_{N_n} - a_{N_0} \right] \quad (20)$$

The final solution for the meridian deflection of the vertical from eqs. (14) and (20) is

$$\xi_i = \left( \xi_0 + \bar{\phi}_{E_i} - \frac{\gamma_i}{g} \right) + \frac{a_{N_i} - a_{N_0}}{g} - \frac{t_i}{t_n} \frac{a_{N_n} - a_{N_0}}{g} + \int_0^{t_i} (\gamma - \bar{\gamma}) dt - \frac{t_i}{t_n} \int_0^{t_n} (\gamma - \bar{\gamma}) dt \quad (21)$$

Equation (21) may be supplemented by

$$\delta \xi_i = \delta \xi_0 + \frac{t_i}{t_n} (\delta \xi_e - \delta \xi_0) - \left( \phi_{S_i} - \frac{t_i}{t_n} \phi_{S_n} \right) \quad (22)$$

to account for astrogeodetic deflection errors and accelerometer bias errors. For a straight traverse, the last term in eq. (22) tends to cancel out.

The rms deflection error  $\sigma_\xi(t_i)$  can be computed by covariance analysis involving the terms without parentheses in eq. (21). Under inclusion of the first two terms of eq. (22),

$$\text{var } \xi_i = \text{var } a_i + \text{var } \gamma_i + \left( 1 - \frac{t_i}{t_n} \right)^2 \text{var } \xi_0 + \left( \frac{t_i}{t_n} \right)^2 \text{var } \xi_n, \quad (23)$$

where  $\text{var } a_i$  is the accelerometer-induced variance and  $\text{var } \gamma_i$  designates the gyro-induced variance. Under consideration of present RGSS parameters, identified in section 1, the rms deflection error  $\sigma_\xi$  is approximately presented in Figure 1 for a vehicle speed of 30 km/h.

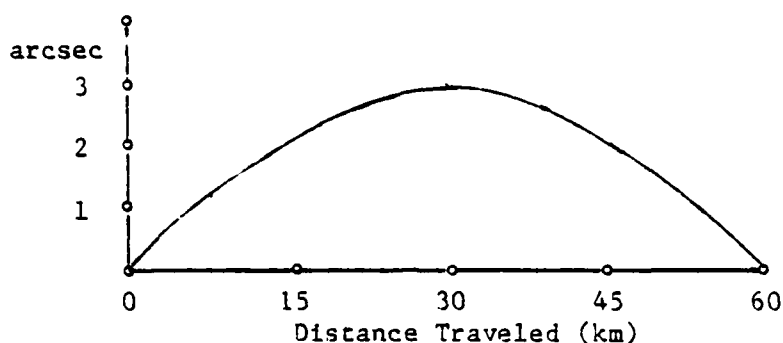


Figure 1

Approximate RMS Deflection Error as a Function of Travel Distance

The variation of normal gravity  $g$  with altitude is not critical in terrestrial applications. The scale factor-generated error does not vanish with respect to L-shaped traverses. It is, however, relatively small for travel times not exceeding 2 hours. Heading sensitivity induced by significant azimuth changes appears to be somewhat critical and may require empirical corrections. Schwarz [7] discussed the impact of heading sensitivity on positioning and concluded that error reductions may be achieved by a modified smoothing procedure. The problem is more intricate in the case of deflection determinations. The use of static accelerometer measurements in conjunction with initial and terminal deflections together with a simplified Kalman filter was first discussed by Huddle [8]. An advanced method for single channel vertical deflection determination was presented by Baussus von Luetzow [9]. It utilizes all observations simultaneously and appears in a modified form in section 4.

4. COUPLED HORIZONTAL CHANNEL OPTIMAL DETERMINATION OF DEFLECTIONS OF THE VERTICAL IN SEMI-FLAT TERRAIN. Higher accuracy requirements necessitating installation of gyroscopes and accelerometers with small error variances and short correlation times as performance characteristics require the integration of the system of differential equations for best possible estimation of  $\eta$  and  $\xi$  and for error assessment in the case of single channel solutions. For this purpose, eqs. (7) and (8) are written as

$$\ddot{x} = S_N \phi_z - g \phi_N + g(\eta - \eta_0) + a_E - a_{E_0} \quad (24)$$

$$\ddot{y} = -S_E \phi_z + g \phi_E - g(\xi - \xi_0) + a_N - a_{N_0} \quad (25)$$

In this way, initial accelerometer calibrations have been taken into account and the initial conditions are

$$\phi_E(o) = \phi_N(o) = \phi_z(o) = \dot{x}(o) = \dot{y}(o) = x(o) = y(o) = 0 \quad (26)$$

The accelerations  $S_N$  and  $S_E$  can probably be neglected. If the system is treated as one with constant coefficients as a good approximation, a closed solution as a function of time is possible. Because of intermittent Kalman filter corrections and the need for numerical weight factors it appears to be advantageous to attempt a numerical solution for  $x$  and  $y$  under utilization of terminal deflection and azimuth data. For economic reasons it is assumed that the survey vehicle travels approximately at a constant speed when in motion and stops every 3 minutes for 1 minute. The speed should not exceed  $10 \text{ msec}^{-1}$  in order to restrict the length of travel intervals.

Solutions for  $\phi_E, \phi_N, \phi_z, \dot{x}, \dot{y}, x, y$  are obtained in accordance with the integration schemes

$$F_1 = F_0 + \dot{F}_0 \Delta t \quad (27)$$

$$F_v = 2F_{v-1} - F_{v-2} + (\Delta t)^2 \ddot{F}_{v-1} \quad (28)$$

with  $\Delta t = 30 \text{ sec}$  and possibly  $60 \text{ sec}$ . The solution structure at the end of the first stop interval, indicated by the subscript  $s=1$ , is

$$F_{v1} = \sum_1^v a_v v_v + \sum_1^v b_v w_v + \sum_1^v c_v \alpha_v + \sum_1^v d_v \beta_v + \sum_1^v e_v \gamma_v \quad (29)$$

In eq. (29),  $v = g(\eta - \eta_0) + a_E - a_{E_0}$ ,  $w = -g(\xi - \xi_0) + a_N - a_{N_0}$ .

Under consideration of Kalman filter tilt corrections, assumed for simplicity to eliminate the integrated first two terms in eq. (9) and the first term in both eq. (10) and (11),

$$F_{v1}^{(2)} = \sum_1^v A_v v_v + \sum_1^v B_v w_v + \sum_1^v C_v \alpha_v + \sum_1^v D_v \beta_v + \sum_1^v E_v \gamma_v + T_1, \quad (30)$$

where  $T_1$  and subsequent  $T_s$  represents an aggregate of tilt-induced random errors. During the stop interval, eqs. (9)-(11) are integrated with  $\dot{x} = \dot{y} = \dot{y} = 0$  with a resultant effect on eqs. (7) and (8). The integration is then continued and yields under utilization of average biases the solution structure

$$F_{vs}^{(2)} = \sum_{v=1}^v A_v v_v + \sum_{v=1}^v B_v w_v + \hat{C}_v \bar{\alpha} + \hat{D}_v \bar{\beta} + \hat{E}_v \bar{\gamma} + R_{vs} + T_s \quad (31)$$

where  $R_{vs}$  designates residual, random-type terms including  $\bar{\alpha} - \alpha_v$ ,  $\bar{\beta} - \beta_v$ , and  $\bar{\gamma} - \gamma_v$ .

At the termination of the survey, when  $\eta_n$ ,  $\xi_n$ , and  $A_n$  are available, it is possible to solve for the gyro biases in the form

$$\begin{bmatrix} \bar{\alpha} \\ \bar{\beta} \\ \bar{\gamma} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} \quad (32)$$

where the  $F$ 's are computable, the  $\epsilon$ 's are an aggregate of  $v$  and  $w$ -errors, and the  $\delta$ 's are aggregates of gyro random and tilt errors. Substitution of  $\bar{\alpha}$ ,  $\bar{\beta}$ ,  $\bar{\gamma}$  from eq. (32) in eq. (31) and separation of  $v$  and  $w$  into deflection and accelerometer errors leads, under restriction to solutions for  $x$  and  $y$ , to the final results

$$\eta_s + \sum_{v=0}^n k_v \eta_v + \sum_{v=0}^n l_v \xi_v = \eta_0 + g^{-1} x_s^{(2)} + \phi_N^{(2)} + r_{gs} + r_{as} + r_{ts} + r_{ds} \quad (33)$$

$$\xi_s + \sum_{v=0}^n m_v \eta_v + \sum_{v=0}^n n_v \xi_v = \xi_0 + g^{-1} y_s^{(2)} + \phi_{Es}^{(2)} + p_{gs} + p_{as} + p_{ts} + p_{ds} \quad (34)$$

The last 4 terms on the right side of both eqs. (33) and (34) are aggregates of random errors associated with gyros, accelerometers, tilt corrections, and initial and terminal deflection errors. Equations (33) and (34) are reformulated as

$$\eta_s + \sum_{v=0}^n k_v \eta_v + \sum_{v=0}^n l_v \xi_v = S_{\eta_s} = M_{\eta_s} - N_{\eta_s} \quad (35)$$

$$\xi_s + \sum_{v=0}^n m_v \eta_v + \sum_{v=0}^n n_v \xi_v = S_{\xi_s} = M_{\xi_s} - N_{\xi_s} \quad (36)$$

where  $S$ ,  $M$ ,  $N$  denote signal, measureable message, and non-measurable noise, respectively.

Under utilization of deflection of the vertical covariance functions,

the prime deflection of the vertical can be optimally estimated in the form

$$\eta_e = A_{ie} [S_{\eta_i} + N_{\eta_i}] + B_{ie} [S_{\xi_i} + N_{\xi_i}] \quad (37)$$

where  $i=s$ ,  $A_{ie}$  and  $B_{ie}$  are matrices of regression coefficients, and the terms in brackets are message matrices. With  $k = 0, 1, \dots, n$ ,  $A_{ie}$  and  $B_{ie}$  can be determined from the equations

$$\overline{\eta_e [S_{\eta_k}]} = A_{ie} [\overline{S_{\eta_i} S_{\eta_k}} + \overline{N_{\eta_i} N_{\eta_k}}] + B_{ie} [\overline{S_{\xi_i} S_{\eta_k}} + \overline{N_{\xi_i} N_{\eta_k}}] \quad (38)$$

$$\overline{\eta_e [S_{\xi_k}]} = A_{ie} [\overline{S_{\eta_i} S_{\xi_k}} + \overline{N_{\eta_i} N_{\xi_k}}] + B_{ie} [\overline{S_{\xi_i} S_{\xi_k}} + \overline{N_{\xi_i} N_{\xi_k}}] \quad (39)$$

In eqs. (38) and (39), where the bar symbol stands for covariance, the noise covariances need only be computed once.

Simplified solutions, particularly in the case of approximately straight traverses, are possible. The coupling of eqs. (33) and (34) should, however, always be considered. If averaged message-type data from repeated surveys are employed, the instrument-generated noise covariances in eqs. (38) and (39) are to be reduced.

5. INTERPOLATION OF GRAVITY ANOMALIES AND DEFLECTIONS OF THE VERTICAL IN MOUNTAINOUS TERRAIN. It is well known that an accurate analytical representation of free-air anomalies in pronounced mountainous terrain can only be achieved by a polynomial of high degree by means of  $\Delta g$ -data available in a network of high resolution. As a consequence, satisfactory linear interpolation requires small mesh sizes  $\Delta x$ ,  $\Delta y$ . It is possible to write the gravity anomaly as

$$\Delta g = \Delta g_i + C_t + r \quad (40)$$

where  $\Delta g_i$  is the isostatic anomaly valid for the compensated geoid,  $C_t$  represents the aggregate of terms computable from the known topography, and  $r$  is a random-type error. In a more general form, also applicable to the optimal estimation of vertical deflections, eq. (40) is reformulated as

$$m = s + n + r. \quad (41)$$

3. For details see Baussus von Luetzow [10]

In this equation,  $m$  is a "message" variable,  $s$  is a "signal" variable,  $n$  is deterministic or computable "noise," and  $r$  is random-type noise.

Under consideration of a linear signal estimation structure, a signal can then be optimally estimated as

$$\hat{s}_e = L(m_i - n_i - r_i) \quad (42)$$

where  $L$  denotes a linear operator and the subscripts  $e$  and  $i$  refer to the estimation point  $P_e$  and measurement points  $P_i$ , respectively. The optimal measurement at  $P_e$  results in

$$\hat{m}_e = \hat{s}_e + n_e + r_e = L(m_i - n_i) + r_e - L(r_i) + n_e. \quad (43)$$

The estimation error is

$$e(\hat{m}_e) = e(\hat{s}_e) + e[r_e - L(r_i)] \quad (44)$$

The corresponding estimation error resulting from the utilization of topographically unmodified measurements  $m_i$  is

$$\begin{aligned} e(m_e) &= \hat{s}_e - L(s_i) + r_e - L(r_i) + n_e - L(n_i) \\ &= e(\hat{s}_e) + e[r_e - L(r_i)] + n_e - L(n_i). \end{aligned} \quad (45)$$

Comparison of eq. (45) with eq. (44) shows that the non-optimal interpolation process is associated with a "topographic" estimation error  $n_e - L(n_i)$  which becomes in general intolerable in moderate to rough mountainous terrain and thus induces the requirement of a fine mesh data grid.

Baussus von Luetzow [10] obtained the following solution for vertical deflections:

$$\begin{aligned} \left\{ \begin{matrix} \xi \\ \eta \end{matrix} \right\} &= \frac{R}{4\pi G} \iint_{\sigma} [\Delta g + G_1(\Delta g)] \left\{ \begin{matrix} \cos \alpha \\ \sin \alpha \end{matrix} \right\} \cdot \frac{dS(\psi)}{d\psi} d\sigma - \frac{\Delta g + G_1(\Delta g)}{G} \left\{ \begin{matrix} \tan \beta_1 \\ \tan \beta_2 \end{matrix} \right\} \\ &+ \frac{R}{4\pi G} \iint_{\sigma} [\delta g_1 + \delta g_2 + G_1(\delta g_3)] \left\{ \begin{matrix} \cos \alpha \\ \sin \alpha \end{matrix} \right\} \frac{dS(\psi)}{d(\psi)} d\sigma + \left\{ \begin{matrix} \delta \xi \\ \delta \eta \end{matrix} \right\} \\ &+ \frac{C - \delta g_2 - G_1(\delta g_3)}{G} \left\{ \begin{matrix} \tan \beta_1 \\ \tan \beta_2 \end{matrix} \right\} \end{aligned} \quad (46)$$

In this equation,  $G$  denotes global mean gravity,  $\Delta g$  is a signal variable profitably synonymous with  $\Delta g_1$  in eq. (40),  $\alpha$  is the azimuth angle

counted clockwise from north,  $\psi$  is the angle between the radius vectors  $\vec{r}_A$  and  $\vec{r}_P$  originating at the earth's spherical center in accordance with figure 2;  $S(\psi)$  is Stokes' function,  $\beta_1$  and  $\beta_2$  are northern and eastern terrain inclinations, respectively,  $G_1$  is a specified integral function,  $\delta g_1, \delta g_2, \delta g_3$  are quantities computable from the earth's topography,  $C$  is the terrain correction, and  $\sigma$  indicates the unit sphere.

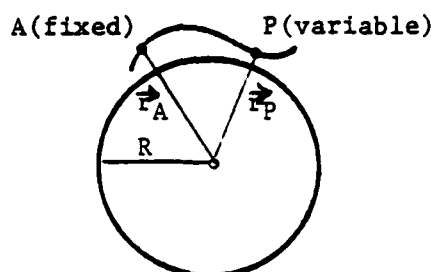


Figure 2

The first two terms of eq. (16), involving the anomaly  $\Delta g$ , represent the "signal" components of  $\xi$  and  $\eta$ . The following three terms constitute computable topographic "noise". Permitting random-type errors  $r_\xi$  and  $r_\eta$ , eq. (46) can be written in analogy with eqs. (40) and (41) as

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \hat{\xi} \\ \hat{\eta} \end{bmatrix} + \begin{bmatrix} \delta \xi_t \\ \delta \eta_t \end{bmatrix} + \begin{bmatrix} r_\xi \\ r_\eta \end{bmatrix} \quad (47)$$

The numerical determination of the three topographic terms of eq. (46) is a complex task, which can, however, be accomplished without inherent difficulties by means of high-speed computers. The estimation of  $\xi$  and  $\eta$  from a set of given vertical deflections can be achieved by means of spatial covariance functions. It is clear from the above analysis that the optimal estimation method outlined in section 4 requires the simultaneous consideration of eq. (47). Otherwise, the linear aggregates involving  $\xi$  and  $\eta$  in eqs. (35) and (36) are to be estimated with some error from a limited set of  $\xi$  and  $\eta$  in order to obtain solutions without linear aggregates. Repetitive surveys would reduce the effects of instrument errors. In summary, the determination of deflections of the vertical in strongly mountainous terrain requires a much increased computational effort to obtain minimum error accuracies. Without such effort, there is a degradation in accuracy.

#### 6. UTILIZATION OF GENERATED GRAVITY VECTOR COMPONENT INFORMATION.

Rapid economical gravity vector component determination is necessary, useful, or potentially useful for the following purposes:



(a) Transformation of astronomic coordinates and astronomic azimuths into ideal terrestrial-geodetic coordinates and geodetic azimuths, requiring knowledge of absolute deflections of the vertical, indispensable for long range ballistic missiles.

(b) Establishment of a three-dimensional anomalous gravity potential  $T$  or analytical upward continuation of first order derivatives of  $\Delta g$ ,  $n$ , and  $\xi$  from discrete measurements thereof at the earth's surface, to extend over relatively large areas, indispensable for long range ballistic missiles with inertial guidance.

(c) Adjustment of geodetic networks.

(d) Geophysical prospecting including estimation of the derivative  $\frac{\partial \Delta g}{\partial r} = 2GR^{-1}N + GR^{-1}\xi \tan \phi - G \left( \frac{\partial \xi}{\partial x} + \frac{\partial n}{\partial y} \right)$  in semi-flat terrain where  $N$  stands for geoidal undulation or height anomaly.<sup>4</sup>

(e) Determination of geoidal differences on land.

(f) Construction of regional gravity anomaly and vertical deflection networks with mesh sizes  $\Delta x = \Delta y$  of about 5 Km as input information for highly accurate gravity programmed inertial surveying systems.

(g) Accuracy improvement of medium range ballistic missiles in conjunction with meteorological corrections and of cruise missiles by utilization of vertical deflection data in accessible areas.

7. CONCLUSIONS. The determination of vertical gravity vector components or gravity anomalies by means of inertial and boundary data has been very promising as to rapidity and accuracy. High accuracy estimation of horizontal gravity vector components or deflections of the vertical from inertial and boundary data requires the installation of gyroscopes, accelerometers, and velocity quantizers with small error variances and short correlation times, the development of sophisticated estimation methods consistent with an outlined structure, and, probably, a three-dimensional Kalman filter. Deflection accuracy in strongly mountainous terrain is degraded. For this reason, a maximum rms error requirement of about 0.3 arcsec in both semi-flat and mountainous terrain may only be achieved by a gradiometer-aided inertial system. Such

4. See Telford et al [11], pp. 7-104, for geophysical applications and Heiskanen and Moritz [12], p. 117, for the equation.

a system would also be beneficial for the location of smaller subterraneous mass anomalies. An effective estimation of gravity anomalies and deflections of the vertical in mountainous terrain from data at points separated by distances of the order 5-10 Km is possible with considerable computational effort under utilization of the developed equations and employment of spatial covariance functions. Numerous military and non-military applications of gravity vector component information generated by inertial and auxiliary data do exist. Gradiometer-aided inertial systems offer additional advantages.

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